



Examination Answer Books

12 Pages

Instructions to Candidates

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.

Complete, fold and seal
(see note 2 below)

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1	Student Number	1707681/1
2	Title of Examination Paper	MATHEMATICS TEST 2
3	Module Code	GOAE4006
4	Date and Time of Examination	11-5-2018 14:00
5	Examination Venue	MAIN HALL COWEN COLLEGE
6	Seat Number	6
7	Number of Answer Books handed in	1

List the numbers of the questions you have attempted	Office Use
Q1	
a	4
b	1
c	0
d	8
Q3	
a	5
b	8
c	7
d	1
e	1
f	7
Total	42/80

Test 2

52.5 %

$$a) AB = \begin{pmatrix} 3 & -2 & -3 \\ 5 & 4 & 1 \\ 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 2 & 1 & 4 \\ 8 & 7 & 3 \\ -5 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 6 & 5 & 8 \\ -16 & 28 & 9 \\ 15 & 0 & 4 \end{pmatrix}$$

$$ii) BA = \begin{pmatrix} 2 & 1 & 4 \\ 8 & 7 & 3 \\ -5 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3 & -2 & -3 \\ 5 & 4 & 1 \\ 2 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 6 & -16 & 15 \\ 5 & 28 & 0 \\ 8 & 9 & 4 \end{pmatrix}$$

iii) Not possible

iv) Not possible. $-24 + 9 - 40 = -55$

$$v) |A| = \begin{vmatrix} 3 & -2 & -3 \\ 5 & 4 & 1 \\ 2 & 3 & 4 \end{vmatrix} \begin{vmatrix} 3 & -2 \\ 5 & 4 \\ 2 & 3 \end{vmatrix} = -1 + 55 = \underline{\underline{54}}$$

$$48 - 4 - 45 = -1$$

$$-140 + 0 + 8 = -132$$

$$vi) |B| = \begin{vmatrix} 2 & 1 & 4 \\ 8 & 7 & 3 \\ -5 & 0 & 1 \end{vmatrix} \begin{vmatrix} 2 & 1 \\ 8 & 7 \\ -5 & 0 \end{vmatrix} = -1 + 132 = \underline{\underline{131}}$$

$$14 - 15 + 0 = -1$$

$$vii) A+B = \begin{pmatrix} 3 & -2 & -3 \\ 5 & 4 & 1 \\ 2 & 3 & 4 \end{pmatrix} + \begin{pmatrix} 2 & 1 & 4 \\ 8 & 7 & 3 \\ -5 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 6 & 6 \\ 6 & 11 & 6 \\ -8 & 1 & 5 \end{pmatrix}$$

viii) $C+D =$ Not possible

4 ✓

$$b) \begin{pmatrix} 1 & 2 & 3x+2y \\ 0 & 4 & 4 \\ 2 & 1 & 13 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 10 \\ 0 & 4 & 4 \\ 2 & 1 & 4x+3y \end{pmatrix}$$

Solve for x and y

$$3x+2y = 10$$

$$4x+3y = 13$$

$$c) \begin{pmatrix} (x+1)-5 & -6 \\ -1 & 4 & 2 \\ -3 & 2 & (x+1) \end{pmatrix} \begin{pmatrix} (x+1)-5 \\ -1 & 4 \\ -3 & 2 \end{pmatrix}$$

$$+30 + 12 = ?$$

d) Cramers Rule

$$\begin{aligned} 3x+2y-z &= 17 \\ 2x-y+4z &= -5 \\ 5x-y-z &= 6 \end{aligned} \quad \begin{pmatrix} 3 & 2 & -1 \\ 2 & -1 & 4 \\ 5 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 17 \\ -5 \\ 6 \end{pmatrix}$$

$$5-12-4 = -11$$

$$D = \begin{vmatrix} 3 & 2 & -1 \\ 2 & -1 & 4 \\ 5 & -1 & -1 \end{vmatrix}$$

$$D = 45 + 11 = 56$$

$$3 + 40 + 2 = 45$$

$$6 - 68 + 10 = -52$$

$$D_1 = \begin{vmatrix} 17 & 2 & -1 \\ -5 & -1 & 4 \\ 6 & -1 & -1 \end{vmatrix}$$

$$D_1 = 60 + 52 = 112$$

$$17 + 48 - 5 = 60$$

$$25 + 72 - 34 = 63$$

$$D2 \begin{pmatrix} 3 & 17 & -1 \\ 2 & -5 & 4 \\ 5 & 6 & -1 \end{pmatrix} \begin{pmatrix} 3 & 17 \\ 2 & -5 \\ 5 & 6 \end{pmatrix}$$

$$D2 = 343 - 63 = \underline{\underline{280}} \quad \textcircled{1}$$

$$15 + 340 - 12 = 343$$

$$-85 + 15 + 24 = -46$$

$$D3 \begin{pmatrix} 3 & 2 & 17 \\ 2 & -1 & -5 \\ 5 & -1 & 6 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 2 & -1 \\ 5 & -1 \end{pmatrix}$$

$$D3 = -102 + 46 = \underline{\underline{-56}} \quad \textcircled{1}$$

$$-18 - 50 - 34 = -102$$

$$D = 56$$

$$\textcircled{D1} = \frac{112}{56} = \underline{\underline{2}} \quad \textcircled{x}$$

$$\textcircled{D2} = \frac{280}{56} = \underline{\underline{5}} \quad \textcircled{y}$$

$$\textcircled{D3} = \frac{-56}{56} = \underline{\underline{-1}} \quad \textcircled{z}$$

P.T.O. →

$$a) Z_1 = 5 + 3j, Z_2 = 2 - 7j$$

$$i) Z_1 + Z_2 = 5 + 3j + 2 - 7j = \underline{7 - 4j} \quad \checkmark \quad \textcircled{1}$$

$$ii) Z_1 - Z_2 = 5 + 3j - 2 - 7j = \underline{3 - 4j} \quad \times$$

$$iii) \frac{Z_1}{Z_2} = \frac{5 + 3j}{2 - 7j} \times \frac{2 + 7j}{2 + 7j} = \frac{10 + 35j + 6j + 21j^2 (x-1)}{4 + 14j - 14j - 49j^2 (x-1)}$$

$$= \frac{-11 + 41j}{54} = \frac{-11}{54} + \frac{41j}{54}$$

$$iv) (Z_1)^2 \times (Z_2) = (5 + 3j)(5 + 3j)(2 - 7j) \quad \textcircled{1}$$

$$= (25 + 15j + 15j + 9j^2 (x-1)) \checkmark$$

$$= (16 + 30j)(2 - 7j) \quad \textcircled{1}$$

$$= (32 + 112j + 60j + 210j^2 (x-1))$$

$$= \underline{-178 + 172j} \quad \times$$

$$b) 12x^2 + 7 = -5x$$

$$12x^2 + 5x + 7 = 0$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \therefore \frac{-5 \pm \sqrt{5^2 - 4 \times 12 \times 7}}{2 \times 12} \quad \checkmark \quad \textcircled{1}$$

$$\frac{-5 \pm \sqrt{25 - 336}}{24} \quad \frac{-5 \pm \sqrt{-311}}{24} \quad \checkmark \quad \textcircled{1}$$

$$\frac{-5 \pm \sqrt{+311} \sqrt{-1}}{24} \quad \frac{-5 \pm 17.63j}{24} \quad \checkmark$$

$$x = \frac{-5 \pm 17.63j}{24} \quad \checkmark \quad \textcircled{1}$$

Express in Polar form

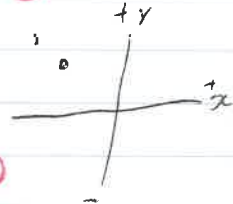
$$\text{(+)} \quad \frac{-5}{24} = -0.2 \quad \frac{17.63}{24} = 0.73$$

$$\text{(+)} \quad r = \sqrt{(-0.2)^2 + (0.73)^2} = 0.75$$

$$\theta = \tan^{-1}\left(\frac{0.73}{-0.2}\right) = -74.67^\circ$$

$$180 - 74.67 = 105.33^\circ$$

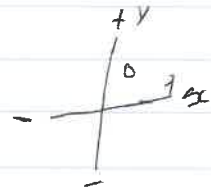
$$0.75 / 105.33^\circ$$



$$\text{(-)} \quad r = \sqrt{(-0.2)^2 + (0.73)^2} = \sqrt{-0.04 - 0.53} = \sqrt{-0.49} \quad \text{X}$$

$$\theta = \tan^{-1}\left(\frac{-0.73}{-0.2}\right) = 74.67^\circ \quad \text{X}$$

$$0.7 / 74.67^\circ \quad \text{X}$$



$$\text{c) i) } (5 + j7)^6$$

$$r = \sqrt{5^2 + 7^2} = 8.6$$

$$\theta = \tan^{-1}\left(\frac{7}{5}\right) = 54.46^\circ$$

$$= (8.6)^6 \left(\cos 6 \times 54.46^\circ + j \sin 6 \times 54.46^\circ \right)$$

$$404867.2 (0.83 - j0.54)$$

$$335790.77 - 218466.28 j$$

$$\text{ii) } (-1 + j3)^5$$

$$r = \sqrt{(-1)^2 + (3)^2} = 3.16$$

$$\theta = \tan^{-1}\left(\frac{3}{-1}\right) = -71.56^\circ$$

$$(3.16)^5 (\cos 5 \times -71.56^\circ + j \sin 5 \times -71.56^\circ)$$

$$315.09 (0.99 + j0.03)$$

$$311.93 + 9.45 j$$

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d) Cartesian form ✓ ①

e) $5 + 3j$ ✓ ①

f) $z_1 = 2 + 3j$, $z_2 = 3 - 6j$, $z_3 = -1 + j$

$$\begin{aligned} z_1 z_2 z_3 &= (2 + 3j)(3 - 6j)(-1 + j) \\ 6 - 12j + 9j - 18j^2(x-1) &= (24 - 3j)(-1 + j) \quad ① \\ &= -24 + 24j + 3j - 3j^2(x-1) \quad ① \\ &= -21 + 27j \quad \checkmark \quad ① \end{aligned}$$

Express in polar form.

$$\begin{matrix} a & b \\ -21 & +27j \end{matrix}$$

$$r = \sqrt{(-21)^2 + (27)^2} = 34.2 \quad \checkmark \quad ①$$

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{27}{-21}\right) = -52.12^\circ \\ 180 - 52.12 &= 127.88^\circ \end{aligned}$$

$$\underline{\underline{34.2 / 127.88^\circ}} \quad ①$$